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## **Explaining the Term Structure of Interest Rates**

**The GKO Market from 1996 to 1998**

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The project is devoted to the analysis of the term structure of interest rates at the Russian GKO market during the period from 1996 to 1998. The sources of inefficiency of the market operation that led to the failure of the pure expectations hypothesis are analyzed. The model with conditional heteroskedasticity with several regimes for unconditional variance quite well describes the one-month GKO series and captures different behavior of the GKO volatility at the initial and final subperiods that were rich in various political and economic shocks.

**Keywords.** interest rates, government bonds, yield curve, term structure, conditional heteroskedasticity, regimes.

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## NON-TECHNICAL SUMMARY

SNG The world history knows only few examples of a so large-scale and heavy in consequences fall of a financial market of a country as the one that took place in Russia in 1998. The pre-crisis history of the GKO market is interesting for many reasons. First, it is a first attempt to create a civilized market for state bonds in the post-Soviet Russia, by analogy with the U.S. Treasury Bills. Second, the Russian market was the largest among young developing markets of countries with transition economies and the most attractive for foreign investors: their share by the year 1997 had reached 35%. However, this attractiveness had a different origin than the attractiveness of the American market for the T-bills, the interest on which is admitted to be risk-free. All participants of the Russian GKO market realized that such unusually high return on the state bonds is associated with additional risk for a number of reasons. First, there was an issue of only very short – three month – bonds (it was not until the year 1996 that an emphasis began to be placed towards an issue of six month bonds), simultaneously with a steady growth of a volume of borrowings. The Government was in need of monetary resources, and the repayment of already existing issues was done by accommodating new ones. Second, the situation with the balance of payments was not favorable, and during the whole examined period there persisted various political risks.

All this makes the Russian market different from the well investigated financial markets of developed countries and interesting to a researcher. The present work is devoted to the analysis of the term structure of interest rates in the GKO market during its blossoming: 1996 – 1998. The studying of the dynamics of the yield curve constructed from the daily data from the secondary GKO trades allows one to track how investors were reacting and adapting themselves to a varying political, economic, external and internal situation in the country. The daily yield curve was being shaped, on the one hand, under the influence of the government regulating such parameters as the volume and duration of an issues, the cut-off price at an primary auction, the refinancing rate, as well as the volume of money in circulation and the exchange rate of the national currency. On the other hand, it was developing under the actions of a big number of agents having all information available to them and aiming at maximizing their profits. Frequently their actions were opposite to those that the state was expecting while applying the control levers, as a consequence of the absence of trust to a policy makers.

As a starting point of investigating the dynamics of the term structure of interest rates at the GKO market there was chosen one of the hypotheses explaining the form of the yield curve – the hypothesis of pure expectations. It asserts that the investors, taking into account all available to them information, form the yield curve in such a way that the time arbitrage is impossible. Or, in other words, the long-term interest rates reflect expectations of the short-term interest rates. The hypothesis is tested in two forms, one of which asserts that the market was dominated by speculative investors interested in obtaining profits fast, while the second implies that the majority of investors are interested in longer-term investments and guaranteed profits during maximum possible time.

Before testing the hypothesis, it is necessary to construct the yield curve correctly. In the paper, the method of doing that is described, which allows one to bypass a number of difficulties specific to any emerging market and to obtain the necessary value of the return for an artificially constructed paper for any day and for any term to maturity.

The results of testing lead to several conclusions. First, the second form of the pure expectations hypothesis did not find a confirmation on the Russian data, which is an evidence of a speculative mood of the majority of investors at the Russian market. Second, the evidence in favor of the first form of the hypothesis was obtained only for the investment horizon equal to six weeks, while for shorter terms an opposite result was obtained, that a high return spread in the short run resulted in a decrease of the return of long bonds, not in an increase. This is a consequence of a low trust to the market: a high spread was perceived by investors not as a signal of growth of long bonds in the future, but as a signal of increasing uncertainty in the market, which instantly resulted in falling of long bonds and to a change in the slope of the yield curve from positive to negative.

The reasons of the obtained results were analyzed in detail, an attempt to their improve was undertaken by examining a shorter investment horizon and shorter time interval during which there were no abrupt political or economic shocks. This did not lead to significant changes in results. The dynamics of arbitrage opportunities at different stages of the market development was also considered. The conclusion is that the failure of the pure expectations hypothesis is a logical consequence of problems of the young Russian market: high volatility, political and economic risks, a small share and low liquidity of long papers, infringement of requirements for market efficiency (such as the absence of insider information, absence of the several large players capable of strongly affecting the market). The rest of the work is devoted to more suitable for studying highly speculative and highly volatile markets models with conditional heteroskedasticity. They were helpful in explaining big volatility bursts for daily one-month GKO interest rates that were observed on the initial (corresponding to Yeltsin's elections) and the final pre-crisis subintervals. Two alternative possibilities of constructing models with three regimes of conditional and unconditional variance with dummy variables in the variance and mean equations were considered. A model with three regimes of unconditional heteroskedasticity showed good properties, allowing one to explain different behavior of the interest rates on three examined subintervals, and to estimate for each a variance and average return according to the size of required risk premium. Both parts lead to the same conclusion that the main feature of the Russian market for the examined period was its high riskiness resulting in abnormal episodes in the behavior of the term structure, big volatility bursts in face of slightest uncertainty, and eventually to a default. One can hope that in the new history of Russian GKO that began in 2000, it will be possible to regulate more finely the segments of economy tied to financial markets, to reduce political risks and gradually to win the trust of investors, thus transforming them to real risk-free state obligations.

## 1. INTRODUCTION

This project aims to analyze the term structure of interest at the Russian market of state short-term bonds from 1996 till 1998. This period encompasses the time period at the beginning of which western experts appraised the Russian financial market as “the market number one” among all emerging markets, it ends with the default and crash of August 17, 1998. Unsurprisingly, such short and stormy history of this market’s existence has not yet been completely studied, and investigation of the reasons why there were anomalies at the market at that period still generates interest. Resurrection of the GKO market in 2000 allows one to use the knowledge and results of this research for policy purposes.

With respect to the trade technology, the Russian state short-term bonds (GKO) are a full analogue of American T-Bills. Their history began in 1993. Before the middle of 1995, GKO played an insignificant role, but then became, alongside with the dollar, a main speculative instrument at the Russian market. The prevalence of the speculative function as the cause of the August 1998 default, though the reasons of the crisis themselves had much deeper roots<sup>1</sup>.

In order to study the term structure of interest at the Russian market of state short-term bonds it is necessary to adapt known econometric methods taking account of the Russian market’s peculiarities. Let us list the main difficulties that are specific to the Russian market:

- the bonds have a short period of existence;
- liquidity is lower than overseas; the fact that allows some speculative anomalies to reveal themselves;
- small share of medium-term government bonds and absence of long-term ones;
- investment into the real sector is hardly a real alternative for a Russian investor (there are many reasons for that – first of all, institutional); consequently, traditional macroeconomic models are barely applicable to the Russian financial market.

After the crisis one could often hear about chaotic and unpredictable behavior of the Russian financial market. We think that this is not so. The main purpose of investigation in this project is to reveal and explain the reasons of the abnormal phenomena in the behavior of the GKO term structure.

## 2. LITERATURE REVIEW

There are many hundreds, if not thousands, of articles of theoretical and empirical nature devoted to the term structure of the interest rates and its application to formation of macroeconomic policy in the West. In Russia, due to a short time span of existence of the state bond

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<sup>1</sup> See, for example, Gurvich, Dvorkovich, 1999; Peresetsky, Ivanter, 1999; Barinov, Pervozvansky, Pervozvanskaya, 1999.

market, the question at hand has not been studied in full. Overwhelming majority of papers on the Russian market of state securities is verbally descriptive and deals with general characteristics of the segment of the market – the GKO index or the average return (Peresetsky, Ivanter, 1999; Drobyshevsky, 1999). Among those few devoted to the analysis of the GKO term structure, most noticeable is (Entov, Radygin, Sinelnikov et al., 1998), performed by group of economists of the Institute of Economics in Transition, who undertake a first attempt to evaluate rationality of the behavior of investors at the GKO market from February 1994 to February 1998. Examining the term structure of interest rates and analyzing conformity of forward rates to future spot-rates, the authors came to a conclusion, that at the Russian market the predictive properties of forward rates are much weaker than at the American market. In most cases investors' expectations hardly can be called rational. However, the obtained results do not allow to completely reject the hypothesis according to which the form of the yield curve depends on investors' expectations.

In Rockinger and Urga (2000) the efficiency of the share market in a transition economy was investigated; as a measure of inefficiency the authors take significance of returns' autocorrelation, or, in other words, predictability. It was shown that the Russian market is characterized by significant predictability. However, the obtained result does not allow the authors to draw any conclusion on the efficiency of the market.

Charemza and Kominek (2000) also studied the question of efficiency of financial markets of various countries. As a measure of inefficiency they considered thickness of tails of distributions of returns at each of the markets. The Russian market turned out to have lower efficiency than the markets of the majority the examined countries.

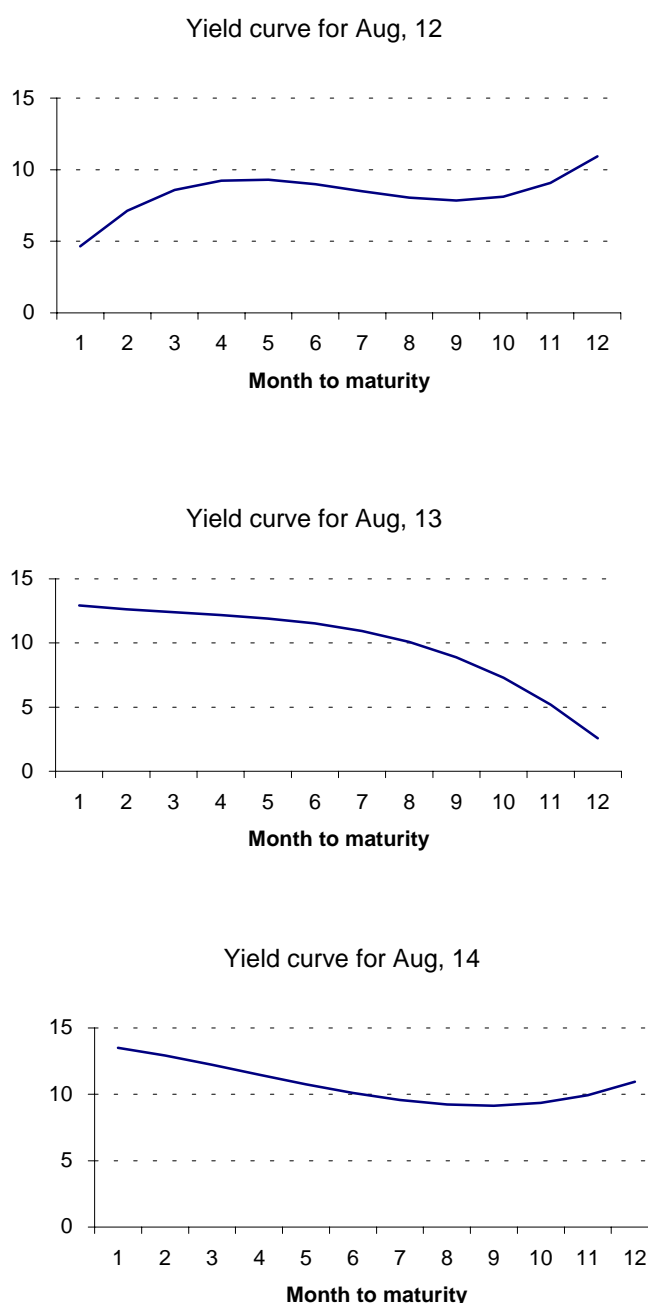
There has been an outburst of work on models with conditional heteroskedasticity for the last two decades, although the threshold autoregression models with conditional heteroskedasticity in the sense we consider here are underinvestigated. The common TARARCH, with a zero threshold for the errors are examined in detail in Rabemananjara and Zakoian (1993). No research on construction of tests and estimation of timing of breaks/thresholds when there are regime shifts in variance take place, to the best of the author's knowledge, is published yet. In a number of papers related topics are mentioned, for example, Perron and Vogelsand (1998) develop tests for a unit roots with a possibility of a break in trend function at an unknown moment of time, but there is no developed universal technique which would suit our case.

It is necessary to also note such works as Gupta and Moazzami (1996) where the authors conclude that in the formation of interest rates the main role play country-specific factors, hence one cannot extend conclusions obtained for one country to others. In del Castillo (1991) it was shown that the presence of significant political risks leads to significant deviations of rates at home markets from the levels implied by the interest rate parity accounting for declared future dynamics of exchange rates. These conclusions once again say that the Russian market, with all its specifics, is in need of a detailed research, and the results obtained for financial markets of other countries, are hardly applicable to it.

### 3. THE PROBLEM FORMULATION

The specifics of the Russian GKO market was in that that frequent changes occurred not only for the overall price level for the state bonds, but also for the term structure of interest rates. The movement of the interest rates was so whimsical that it seemed to be totally circumstantial and unpredictable. Even in the short run the yield curve changes its slope, its convexity or concavity. The graphs below show the behavior of the term structure of the GKO interest rate on the eve of the crisis: the curve abruptly changed its slope, from positive, which is considered normal, to negative, which is typical for a pre-crisis situation.

When interpreting the term structure of interest rates, one most frequently uses three hypotheses: the Pure Expectation theory, the Liquidity Preference theory, and the Market Segmentation theory. The Pure Expectation Theory in its general form states that the long-term interest rates reflect expectations of the short-term ones. Thus, an increasing yield curve is an evidence of rises in spot rates in the future. According to the Liquidity Preference theory, investors demand an additional premium, the larger the longer the period of possession of the paper, because, first, the invested money may be in demand before maturity, and second, short-term investment are less prone to the risk of changes in the interest rate. Finally, the Market Segmentation theory is based on the assumption of the division among investors and borrowers into groups either by preference for an investment term or having to invest into bonds with a particular term because of legislative restrictions. In this case, the spot rate is determined by demand and supply at



**Fig. 1.** The term structure of GKO interest rates on three pre-crisis days.

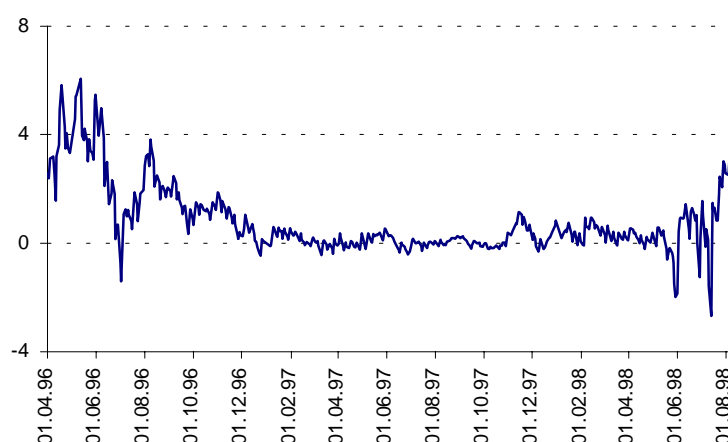


each segment of the market separately, so the rates do not follow the same dynamics. There are other theories – the Time Varying Risk Premium theory, which takes account of influences of exogenous variables on the value and sign of the premium, the Preferred Habitat theory, which considers the market term structure a result of independent decisions by a large number of economic agents. These two theories are essentially varieties of the former three.

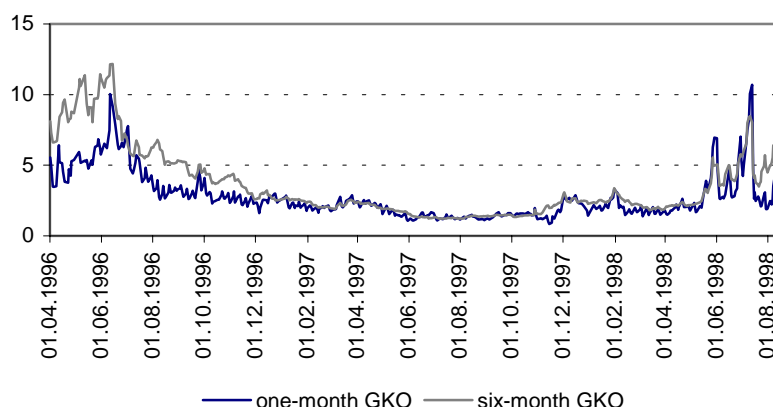
Macroeconomic approaches to the analysis of the term structure of interest rates are directed towards studying the effects of monetary and fiscal policies. As a rule, such models do not describe dynamics of the entire yield curve, but one distinguishes dynamics of the reaction of short- and long-term rates to the state's actions. Turnovsky (1995) presents the analysis of the behavior of the term structure of interest rates from the viewpoint of macroeconomic policy. Empirical testing of models of the term structure on the basis of macroeconomic approaches amount to studying factors that cause fluctuations of the term premium.

The first and mostly empirically tested is the hypothesis of pure expectations. According to this hypothesis, a big spread in bond returns today must be followed by a rise of returns on short-term bonds in the long run and on long-term bonds in the short run. However, a number of empirical tests for markets in different countries revealed similar results: the slope of the term structure almost always gives incorrect prediction for short-run changes of returns on long-term bonds, but correct one for changes of returns on short-term bonds in the long run (Campbell and Shiller, 1991). The authors interpret such behavior of the yield curve as overreaction of the spread of returns: the long-term rate differs from the short-term rate in the direction predicted by the expectations hypothesis, but the spread between the two rates is larger than could be justified by rational expectations of future changes in the short rate.

The graphs presented below show the dynamics of the spread between six- and one-month GKO rates, as well as the rates themselves on one- and six-month GKO in the time period under investigation.



**Fig. 2.** Spread between six- and one-month GKO (% APR).



**Fig. 3.** One- and six-month GKO rates (% APR).

Let us concentrate in more detail on the factors influence of which on the market of the Russian state securities could determine a level of interest rates in general, and also the change of the form of the yield curve. Russian GKO for their entire history of development could not be regarded as risk-free securities, they were rather the tool of an active speculative game.<sup>2</sup> There are a number of reasons why the investors demanded an additional premium for risk when forming the ideas about their required return, and as a consequence about the yield curve. These are changes of expectations due to the risk of inflation, the risk of devaluation (currency risk), the risk of default (country risk). A sudden increase in one or more risks should lead to incorporation of the changed expectations of investors into their demanded term premium, and as a consequence to a rise in the spread between different terms bonds and the slope of the yield curve.

Besides, the reason of the sudden changes in the yield curve slope could be political. For example, the negative slope of yield curve has been during a few weeks before elections of the President of Russia in 1996; during the global financial crisis in 1997; as a result of an interview of the Head of the Central Bank of Russia S. Dubinin in April, 1998, where he declared a possible devaluation of the ruble. etc. These events could also become the reason of the changes in behavior of the term structure of interest rates in Russian market GKO in 1996-1998, that are inexplicable within the framework of classical theories.

However, the answers to the questions listed above to some extent depend on a method to construct the yield curve. Because there is no official methodology in Russia, a question of a choice of the optimal variant remains open. There are objective difficulties of the correct construction of the yield curve under conditions of underdeveloped market. The decision on the way of the construction of the yield curve from the results of daily trades of Russian state papers in the secondary market and the method of processing raw data is also an important preliminary phase of investigation.

<sup>2</sup> Barinov, Pervozvansky, Pervozvanskaya (1999)

Further to explain the term structure behavior I

- test the pure expectations hypothesis (in two forms differing by the type that the majority of investors have);
- consider reasons of violation of PEH;
- try to improve the results or testing the hypothesis by excluding from consideration most rich in political and economical shocks the initial and final periods;
- try to explain volatility jumps by modeling conditional variance;
- explain the different behavior of returns during different time periods by using model with three different regimes of conditional (and unconditional) volatility.

#### 4. RESEARCH METHODOLOGY

In order to avoid the influence of casual and ungrounded changes in the prices and returns of concrete bonds, it is necessary to correctly construct the yield curve. The thing is that the analysis of dynamics of the price of each concrete security is rather involved because of the following reasons:

- bonds' prices grow as the period of redemption approaches (the presence of this positive trend obstructs the comparative analysis of different securities);
- in reality it is impossible to trace the movements in the prices for all securities that circulate in the market;
- the movements in the price of a security in the short run may not comply with the overall market trend; moreover, often small trades for a concrete security are made with anomalous prices (naturally, it would not be logical to take these suspicious trades into consideration).

In order to make the analysis clean, artificial bonds were designed<sup>3</sup>. In contrast to real securities, the artificial GKO is eternal and always has a history. This nonexistent security that reflects in certain degree the returns of all GKO that circulate at the secondary market, is less subject to casual unjustified changes in price and, hopefully, in greater degree depends on conditions of the market and macroeconomic parameters.

In order to construct a virtual GKO, we processed daily data (from April 1, 1996 to August 14, 1998) on GKO trades at the secondary market. The data were taken from the MICEX, CBR Websites. At the first stage of constructing the virtual GKO we excluded the data on those trades, the volume of which did not exceed 0.3% of the overall daily trade volume of a trade session, since there are reasons to think that these are "grey" deals, whose prices are unreasonable to consider a market tendency. We also excluded very short papers with redemption period of less than 6 days,

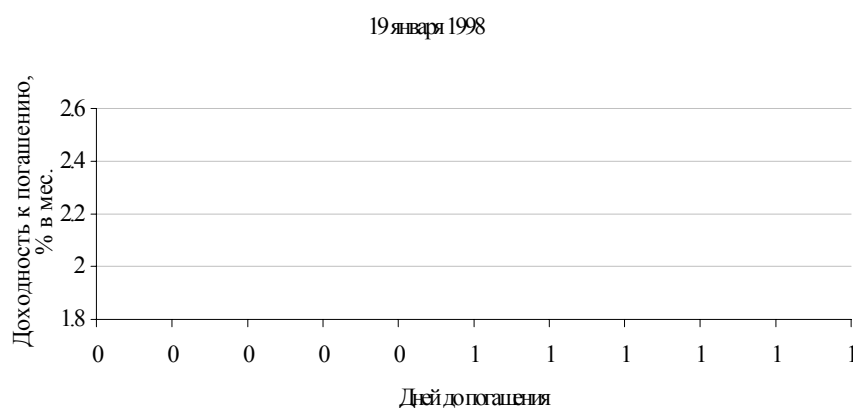
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<sup>3</sup> Bukhvalov, Kryukovskaya, Okulov (2001).

since their returns were determined by an excess demands for them from commercial banks for translation of assets into money resources. At the second stage the returns data sorted in this way were approximated by third-order polynomials.

There exist a variety of approaches and strategies of construction of the yield curves. The classical method of construction of the yield curve was considered by McCulloch (1975). It offered the method of approximation of the data by cubic splines. However, under such way of constructing the constructed yield curve has a large number of bends that do not have economic meaning. Since then various methods have been offered, not always suitable to the specifics of the Russian market.

Entov et al. (1998), for instance, describe the approximation method by linear splines. The drawback of this method is that the returns of papers with different maturities practically do not differ from each other, that is to say, do not reflect in full the dynamics of the term structure. Drobyshevsky (1999) used the method of averaging - a linear interpolation of prices for groups of papers with close maturities. The shortcoming of this method is that in the market there may be no papers with necessary maturity, or, alternatively, there may be too many and with big diversity in prices. For the purpose of the present work it seems to be the optimal the choice of a cubic polynomial: on the time horizon considered (bonds with maturity of 6 months) the yield curve most exactly is approximated by a cubic polynomial (from the viewpoint of minimization of squared deviations). Fig. 4 presents the yield curve built for April 19, 1998.



**Fig. 4.** Yield curve approximated by a cubic polynomial for April 19, 1998.

Thus, we will obtain the term structure of GKO interest rates at any trading day of the considered time period. Taking points on the obtained curve with any term before repayment that is of interest to us, we will obtain time series of returns. However, the high frequency of observations can lead to higher influence of random noise and fluctuations on the market, associated in larger degree with short-term fluctuations in the liquidity of the market, with actions of large players, than with macro-economic factors. The majority of studies of the term structure of interest rates at western markets are based on monthly and quarterly data. We instead have averaged daily returns during a week, since, first, the time series under investigation is too short to consider monthly data, and secondly,

the period under investigation was characterized by a large number of shock situations, which significantly were changing the form of the yield curve. Thus by averaging during a month we would have lost important information kept in the dynamics of yield curve.

## **5. PURE EXPECTATIONS HYPOTHESIS**

### **5.1. Degree of efficiency GKO market**

An important assumption in the pure expectations hypothesis is that of the perfection and efficiency of the bond market: all participants possess full information, information spreads out instantly, and is fully reflected in the prices all market instruments. Thus the prices reflect intuitive views of market participants concerning future prices and returns. This allows, knowing prices of all bond at present, to predict the prices of bonds in the future. Because not always one manages to find a confirmation of efficiency empirically even in the developed markets, it is not serious to speak about efficiency of the GKO market in the full sense of this word. However, the history of the development of the GKO market allows one to assume some degree of weak efficiency in the chosen period (April 1996 – August 1998). The formal date of birth of GKO was May 18, 1993 – the day of the first auction where an allocation of bonds with 3 months maturity took place. However, according to such characteristics as the volume of trades, market capacity, liquidity and dynamism of development, as well as the share in financing the deficit of the state budget, only since 1996 the GKO have taken the leading place among the state obligations of the Russian Federation.

At the first stage of existence of the GKO (1993-1995) the market was characterized by a small volume and strong variability of returns. Increases of the volume of trades and relative stabilization of returns were managed to achieve only in 1996. It is explained by several factors. First, the absence of an organized market and legal regulation in this field was the basis of cautious behavior of potential investors. A high unstable rate of inflation also did not allow the GKO market to develop in this period.

Second, at this stage the trades were conducted at a unique platform - the Moscow Interbank Currency Exchange (MICEx). The second regional platform - the St.-Petersburg Interbank Currency Exchange (SPbICEx) - started trades only from December, 1995. By the end of 1996 the system of inter-regional GKO trades already included eight stock exchanges: the Moscow Interbank Currency Exchange (MICEx), the St.-Petersburg Interbank Currency Exchange (SPbICEx), the Siberian Interbank Currency Exchange (SICEx), the Rostov Interbank Currency Exchange (RICEx), the Ural Currency exchange (UCEx), the Asian-Pacific International Currency Exchange (APICEx), the Nizhniy Novgorod Currency-Stock Exchange (NCSEEx), the Samara Interbank Currency Exchange (SICEx). Although the share of regional stock exchanges in the total amount of trades remained low - on the average about 5% in the volume of daily secondary trades, it does not reduce significance of the formation of the trading and payment system which functions in a real time regime in regions with different time zones.

Finally, in January 1996 there was made the decision on liberalization of the access of non-residents to the GKO market. In August 1996 the Central Bank of the Russian Federation has commissioned a new scheme of the access of foreign investors to the GKO market, which presumed their participation in the trades through the Russian authorized banks - dealers at the secondary trades, with subsequent repatriation of the profits through accounts of the "C" type. The initial quota for non-residents was established at the rate of 1 billion dollars per month, but then it was increased up to 1.5 billion dollars per month, and since November 1996 - up to 2 billion dollars per month. The following change of the operating procedure for non-residents was on January 1, 1998. The restrictions on the term of investment into the bonds and on the period of repatriation of profits were abolished. In addition, the Central Bank transferred to commercial banks the right to insure foreign investors from the currency risk with the help of currency forward contracts.

The listed factors lead to indirect evidence in favor of a certain degree of information efficiency of the GKO market since 1996 to 1998, at least to the degree sufficient to speak of rationality of expectations of market agents.

## 5.2. Two forms of the Pure Expectations Hypothesis

The *Pure Expectations Hypothesis (PEH)* claims that at an efficient market there is no arbitrage between securities of different maturities. Strictly speaking, this hypothesis exists in two mutually exclusive forms (Campbell, MacCinlay and Lo, 1997).

*First form of PEH.* By buying a short security (with redemption in 1 month), the investor expects to get as much as if he/she bought a long security (with redemption in N months) and in 1 month sold it. Mathematically this can be written as follows:

$$(1 + Y_{1,t}) = (1 + Y_{N,t})^N \cdot E_t \{(1 + Y_{N-1,t+1})^{-(N-1)}\}, \quad (1)$$

where  $Y_{1,t}$  – monthly return bond with 1 month to maturity at time t;  $Y_{N,t}$  – monthly return bond with N month to maturity at time t and  $Y_{N-1,t+1}$  – monthly return bond with N-1 month to maturity at time t+1.

*Second form of PEH.* By buying a long security (with redemption in N months), the investor expects to get as much as if he/she bought a short security (with redemption in 1 month) and (N-1) times reinvested the income into a similar short security again. Mathematically, this can be written as follows:

$$(1 + Y_{N,t})^N = E_t \{(1 + Y_{1,t}) \cdot (1 + Y_{1,t+1}) \dots (1 + Y_{1,t+N-1})\} = (1 + Y_{1,t}) \cdot E_t \{(1 + Y_{N-1,t+1})^{N-1}\}. \quad (2)$$

Note that in general the two forms of the *PEH* are not equivalent since

$$E_t \{(1 + Y_{N-1,t+1})^{N-1}\} \neq 1 / E_t \{(1 + Y_{N-1,t+1})^{-(N-1)}\}.$$

The difference between the two formulations reflects a difference in the investors' behavior at the market. In the first case the market is dominated by a speculative mood, the investors are interested in maximum short-term profit. In the second case the investors are interested in a long-term investment and a guaranteed long-term profit. Usually this difference is not taken into attention. However,

the difference between the two formulations causes the situation when relative movements of the interest rates for short and long papers differ significantly from each other not only in size, but also in sign. Consequently, the clarification of which PEH form is appropriate for the Russian market has important practical importance for the investor.

### 5.3. Construction of econometric model

For testing the two versions of the pure expectations hypothesis we will use the econometric model described in Campbell, MacCinlay and Lo (1997). To this end, we define the Yield Spread as the difference in long (for  $N$  period maturity) and short (for one period maturity) returns of the constructed virtual GKO.

$$S_T(N) = Y_T(N) - Y_T(1). \quad (3)$$

However, for statistical calculation it is better to handle, instead of returns themselves, the corresponding natural logarithms of returns, which we will denote by small letters

$$y_T(t) = \ln(1 + Y_T(t)). \quad (4)$$

Then the spread of logarithms of returns  $s_T(N)$  will have a following form:

$$s_T(N) = y_T(N) - y_T(1). \quad (5)$$

Converting formula (1) and substituting the definition of the returns spread (5), we will get the expression

$$E_t \{y_{T+1}(N-1) - y_T(N)\} = s_t(N)/(N-1),$$

on the basis of which it is possible to build an econometric model for the first form of the pure expectations hypothesis

$$y_{T+1}(N-1) - y_T(N) = \alpha(N) + \beta(N)(s_T(N)/(N-1)) + \varepsilon_T(N), \quad (6)$$

where  $\varepsilon_T(N)$  is a random error in observation  $T$ . Having found estimates of the coefficients  $\alpha(N)$ ,  $\beta(N)$  we will be able to test the first form of the pure expectations hypothesis from the returns of the constructed virtual GKO: if the first form of the pure expectations hypothesis exists, the coefficient  $\alpha(N)$  should insignificantly differ from zero, and the coefficient  $\alpha(N)$  should be statistically significant and close to unity.

To obtain an econometric model testing the second form of the pure expectations hypothesis, we will transform formula (2) and substitute the definition of the returns spread (5). The obtained result

$$E_T \left\{ \sum_{j=1}^{N-1} \left[ \left(1 - \frac{j}{N}\right) (y_{T+j}(1) - y_{T+j-1}(1)) \right] \right\} = s_T(N)$$

allows us to write the regression equation

$$\sum_{j=1}^{N-1} \left[ \left(1 - \frac{j}{N}\right) (y_{T+j}(1) - y_{T+j-1}(1)) \right] = \gamma(N) + \delta(N)s_T(N) + \varepsilon_T(N). \quad (7)$$

If this second form of the pure expectations hypothesis is true, the coefficient  $\gamma(N)$  should insignificantly differ from zero, and the coefficient  $\delta(N)$  should be statistically significant and close to unity.

#### 5.4. The results of testing Pure Expectations Hypothesis

However, testing the hypothesis using the entire time interval the results turn out to be rather contradictory: below are the results of testing the hypothesis for  $N$  equal 2, 3 and 6 weeks carried out on the weekly series of GKO returns. Equation was estimated by OLS with Newey-West estimator of covariance matrix consistent in the presence of heteroskedasticity and autocorrelation of unknown form. Testing on the presence ARCH effects in the residuals not gives positive results.

**Table 1.** The results of econometric testing of the PEH.

	N=2	N=3	N=6
$\beta$	-0.808	-0.641	1.106*
$\delta$	0.142	0.116	0.127

\* - significantly close to 1 at 99% level.

As can be seen from the results, a proximity of  $\delta$  to zero is an evidence of invalidity of the second form of the pure expectations hypothesis for the Russian GKO market during the period under consideration, irrespective of the horizon. This comes at no surprise, since the Russian market of government bonds has always been directed towards short papers and attracted those investors interested in fast profits. The results of the testing of the first form of the pure expectations hypothesis are even more interesting. For  $N=2$  the value of the coefficient  $\beta$ , although close to zero, has a negative sign. This is an evidence of the fact that at the Russian GKO market a high returns spread in the short run presumably should lead to a reduction in the returns of long papers, contrary to the predictions of the pure expectations hypothesis. The same result has been repeatedly obtained in the literature when testing the pure expectations hypothesis at financial markets of other countries. For  $N=6$  the rates behave in accord with the first form of the hypothesis – the coefficient is positive and close to unity.

The fact that the coefficient estimates vary so much with the horizon is again evidence that when explaining the form of a yield curve it is necessary to take into account the liquidity hypothesis and the hypothesis of market segmentation. According to these theories, an excess return arising at some segment of the term structure cannot be reduced as a result of an intertemporal arbitrage, as required the rational expectations hypothesis.

However, each of these hypotheses in its own – the liquidity hypothesis and the hypothesis of market segmentation – are also rejected on the basis of Russian data<sup>4</sup>.

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<sup>4</sup> Drobyshevsky, 1999



Yet the main explanation of such results is that the Russian market of government securities is young, and the period under investigation is rich in various shock situations. All this could not help but worsen the predictive characteristics of the yield curve. Let us try to improve the result by excluding from consideration most rich in shocks the initial and final periods and concentrate on the short GKO with maturities of 1, 2 and 3 weeks.

From the point of view historical events, the period under consideration (1996-1998) it is reasonable to single out three subperiods during which one could observe differences in the dynamics of the general level of GKO profitability:

- Before July 1996 года – the period of instability of the market associated with high inflationary and political risks and the development of the GKO-OFZ market;
- August 1996 – November 1997 года – the period of low average level of profitability, characterized by the lowest level of risks and highest liquidity;
- December 1997 – August 1998 года – the period of development of the financial crisis, decrease of the confidence of participants in government securities and maximal involvement of the Central Bank of Russia in supporting the bond prices.

A visual analysis of the behavior of 1, and 6-month GKO (Fig. 3) also allows one to select nearly the same initial and final periods being characterized by a strong volatility, uncertainty at the market that does not allow one to rely on the pure expectations hypothesis during these time intervals. Table 2 presents the results of testing the pure expectations hypothesis for the shortened for the abovementioned reasons time interval (August 1996 – May 1998) for GKO with 2, 3 or 4 weeks to maturity.

**Table 2.** The results of econometric testing of the PEH.

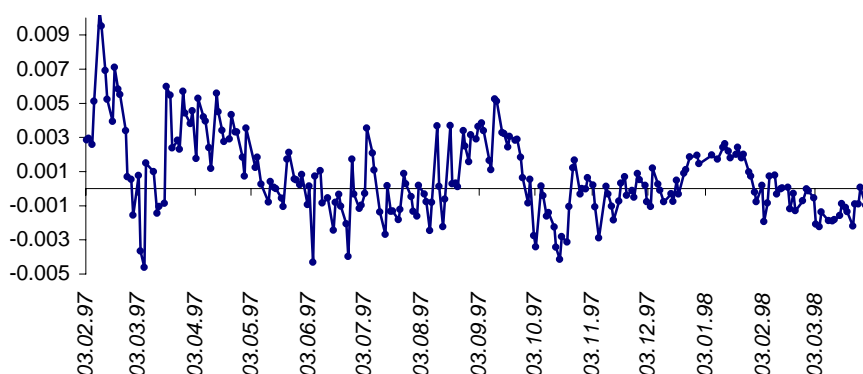
	N=2	N=3	N=4
$\beta$	-0.789	-0.751	-0.711
$\delta$	0.086	0.040	0.0140

As we see, no principle improvement have occurred. The second form of the hypothesis is still rejected, even more strongly than in the previous case, the coefficient  $\beta$  almost does not change as N grows, but, in spite of closeness to unity in absolute value, has a negative sign. Apparently, even during this relatively calm period quite frequently there appeared sharp aggravations of the problem of liquidity, forcing the yield curve have a negative slope (Fig. 5). Moreover, the negative slope kept up sometimes during several weeks, in spite of the relative political and economic tranquility.

It is necessary to note that the evaluation of predictive properties of interest rates on relatively short time intervals is influenced by various “state of the market” factors. It was shown<sup>5</sup> that qualitatively

<sup>5</sup> Mishkin and Fama

better forecasts based on the term structure can be calculated on long-term segments of the yield curve. However, in the circumstances of underdevelopment of the long-term sectors of the Russian market up to 1998 it is impossible to conduct a detailed comparison.



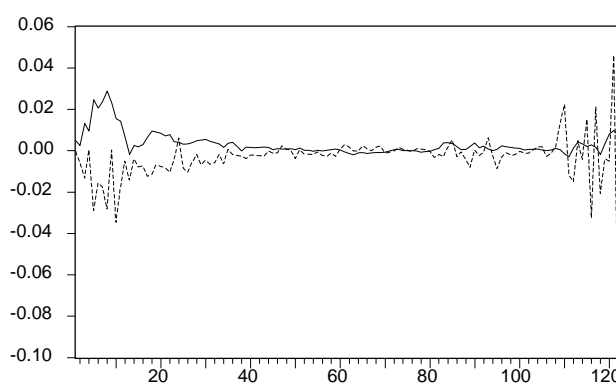
**Fig. 5.** Spread between 6 and 1-month GKO from 01.02.97 to 01.04.98.

The negative value of spread corresponding to negative slope of the yield curve.

### 5.5. Arbitrage opportunities

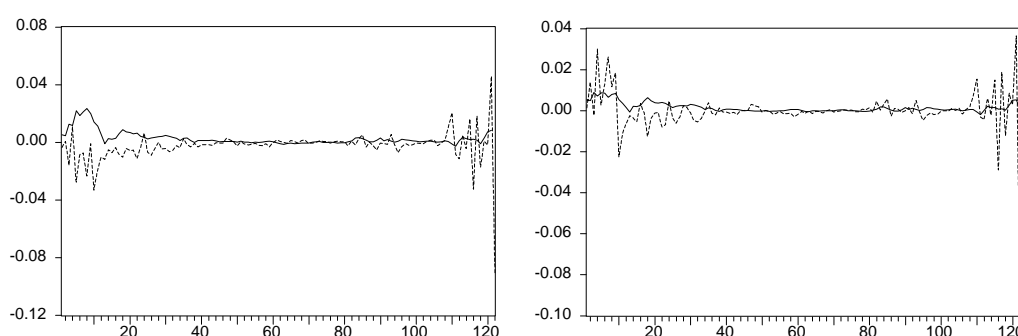
In order to get an idea about arbitrage opportunities during the entire examined interval from 1996 to 1998 we will look at the dynamics of the left and right sides of equation (6) which we considered when testing for the first form of the pure expectation hypothesis (Fig. 6). The continuous line corresponds to the left side, regressand  $y_{T+1}(N-1) - y_T(N)$ , and the dotted one – to the regressor  $s_T(N)/(N-1)$ . If the hypothesis was true, that is, if there existed no intertemporal arbitrage opportunities, the quantity  $\beta(N)$  would have to be (approximately) equal to unity, and both lines would differ from each other only by the size of a random error  $\varepsilon_T(N)$ . On the horizontal axis are weeks since April 1996 till August 1998.

As one can see the picture appreciably differs from the ideal. During the entire period there were arbitrage opportunity. The richest in this sense was the first subperiod – the peak corresponds to the presidential elections (a difference between the two lines, the arbitrage profitability, makes more than 4% per month), after which the arbitrage profitability was gradually going down.



**Fig. 6.** The dynamics of arbitrage opportunities estimated when testing for the first form of the pure expectation hypothesis, with  $N=2$ .

During the course of 1997, together with a big reduction in GKO rates, the arbitrage profitability fell as well, but, judging by frequent changes of the slope of the yield curve, the arbitrage opportunities were in effect constantly. At last, in the last pre-crisis period one can speak rather of funds removed from the falling market in time than of an arbitrage. If there were arbitrage opportunities, they were associated with a too high risk. It is also interesting to note that the longer GKO duration the less their arbitrage attractiveness was, due to the low liquidity. Figure 7 presents the arbitrage opportunities for  $N=3$  and  $N=6$ .



**Fig. 7.** The dynamics of arbitrage opportunities estimated when testing for the first form of the pure expectation hypothesis, with  $N=3$  and  $N=6$ .

Especially appreciable is a reduction in arbitrage opportunities in the first period, for  $N=6$ . For example, up to the presidential elections there was no opportunity to obtain an arbitrage profit in contrast to the case  $N=2$  for the same period. Investors did not want to deal with 6-month papers in a situation of high political uncertainty. These calculations are given for GKO returns averaged weekly, so a part of fluctuations of returns was smoothed. Hence, in consideration of daily returns arbitrage opportunities arose even more frequently.

### 5.6. The reasons of failure of the pure expectations hypothesis

Let us consider the episodes when under the influence of external (economic, political) factors the dynamics of the term structure predicted by the pure expectations hypothesis could be violated. Let us enumerate the most important events of a political and economic nature during the period under investigation that are capable to sharply influence the dynamics of the term structure.

In January 1996 it was decided to liberalize of an access of non-residents to the market. On February 7 foreign investors obtained the right to participate in OFZ auctions through accredited resident banks. Starting from March 1996 the GKO-OFZ market more and more felt the influence of the presidential elections campaign. The necessarily of financing the election campaign and reinforcement of political risks at the same time brought about an increase of the borrowing cost at the market from 50% to 200-250% annually. However, the successful election of B.Yeltsin on June 16, 1996 become a signal to a reduction of political risks, and the interest rates returned to their former

level. In August 1996 there was introduced a new simplified scheme of access of non-residents to the GKO-OFZ market.

Year 1997 became the most tranquil period of the market's history. However, from November 1997 the Russian financial market began to feel the influence of the world financial crisis. The returns of the state bonds began to slowly grow in spite of the fact that the Central Bank of Russia continued to maintain the prices. In December the CB abandoned the policy of maintenance of the rates at low levels. On January 1 there was introduced a new order of non-residents' work at the GKO-OFZ market, which even more simplified the procedure of withdrawal of funds from the Russian ruble nominated securities and thus promoted the market's instability. However, the CB during December 1997–January 1998 was able to withstand the attack on the ruble.

From February to April 1998 the GKO-OFZ market experienced the influx of new funds which allowed the interest rates to remain to be stable at a level of about 30% annually. This led to a further build-up a volume of the state debt. From January to May 1998 the GKO and OFZ volumes in circulation increased from 16.5% to 18.4% of the GDP. On April 1 the Central Bank of Russia chair Sergey Dubinin in an interview to the Financial Times declared a need to bring the rate of ruble devaluation nearer the rate of inflation. According to Dubinin's wording, the ruble by the moment had recovered from the Asian financial crisis and it was necessary to correct its rate. The devaluation aftermath was not carried out, but the news about the CB plans of adjustments of the exchange rate policy had influenced the behavior of investors.

The situation with refinancing and servicing the market obligations by the Russian government became complicated at the end of April 1998 and. The volume of new funds attracted at auctions and when allocating the bond on the secondary market, could not cover the volume of state obligations, but attempts to enlarge the volume of the allocation brought about returns growth of newly issued bonds which even more reduced the real volume of borrowing.

At the end of May the Ministry of Finance of Russia had to abandon the allocation of newly issued ruble nominated bonds, while the funds for redemption of earlier issued series were taken from the federal budget revenues and from allocation of new eurobonds. From June the volume of Russian internal debt became to contract.

In spite of this by mid-July the GKO-OFZ prices had fallen practically to the default level. Simultaneously, a non-residents' fund outflow from the country increased, and the danger of devaluation of the ruble went up. On July 24 the Russian government organized a conversion of a part of the ruble nominated internal debt to eurobonds of amount about 27.5 bln. Rubles using the funds of the IMF stabilization credit, in order to raise the confidence of investors to Russian securities and reduce the level of the exchange rate risk. The returns level at the market by the end of July had fallen to 50–60% annually.

However, all the undertaken measures with the background of continuing deterioration of the situation in the budgetary sphere even more reduced the confidence in abilities of the Russian government to execute their own security obligations in full. From the beginning of August the capital out-

flow increased. By August 14 the returns on papers went up to 100% annual, and the long bond issues practically lost their liquidity. On August 17 the Russian government and CB released a joint statement, in which they declared a cessation of the secondary trades of the state bonds, and on August 19 a default on the following redeemed GKO issue took place.

As we can see (Fig. 5), even the "tranquil" year of 1997 (from the point of view of in-country events) experienced the influence of the global financial crisis (often negative slope of the yield curve). While internal investors considered the market of the Russian state bonds as highly reliable, for non-resident investors it remained one of quite risky markets of lesser-developed countries. The results of a low trust to the state policy were an increased premium for risk, a strong volatility, increased demand for short papers, low liquidity of long paper and consequently often negative slope of the yield curve. Besides, in spite of big volumes of trades, few large agents, such as Sberbank, the Central Bank and Orgbank, formed more than half of them. That is infringement of one of basic conditions for an efficient market, namely actions of a plenty of independent agents having equal strength. Thus, the failure of the pure expectations hypothesis is a logical consequence of all listed features of the GKO market in the considered time period.

## 6. MODELING CONDITIONAL VARIANCE

By averaging the GKO returns during weeks while testing the pure expectations hypothesis we lost a part of the information on the dynamics of returns. As we saw, such averaging has not resulted in evidence in favor of the hypothesis, therefore we will try to consider now the daily data on the GKO returns and to analyze their behavior. We are interested in the dates of hikes of the volatility. The literature has repeatedly noted the connection between inefficiency of a financial market and the "thickness" of tails of the distribution of returns<sup>6</sup>. Theoretically, in the case when a large number of independent agents operate in the market, the normal distribution of returns should take place. In practice the requirement of normality is never carried out, but there is a tendency that the less degree of efficiency of the market the "thicker" the tails of the distribution of returns is. For modeling financial time series having the "heavy tails" property one most often uses models with conditional heteroskedasticity. Let us consider several alternative specifications for the processes of the conditional variance of the errors. Table 3 presents the results of estimation the most successful models.

Coming from the presented in the table results of estimation (standard errors in brackets), the best model among the considered ones turns out to be AR(3)–ARCH(1,1). Although the information criteria clearly select the last model – AR(3)–TARCH(1,1) with the lagged spread variable  $\text{Spred}(-1)$  added to the variance equation, the coefficient at this variable is practically zero, and the sum of coefficients at ARCH(1) and  $(\varepsilon_t < 0) \cdot \text{ARCH}(1)$  terms is negative.

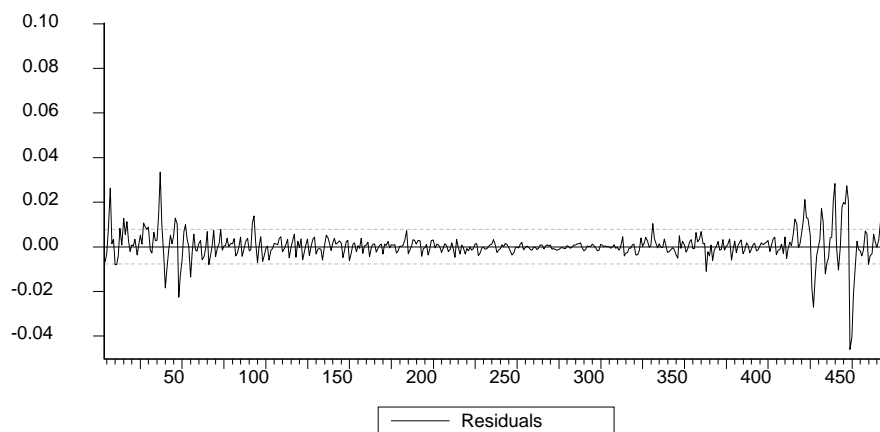
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<sup>6</sup> Charemza W.W., Kominek Z. "Thick tails and speculative processes: a simulation and empirical analysis", Working Paper, University of Leicester, 2000.

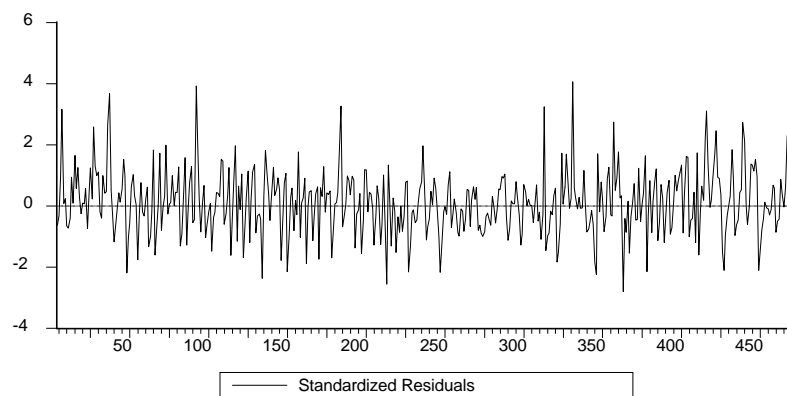
**Table 3.** Estimation results.

Model	ARCH(1)	AR(2) - GARCH(1,1)	AR(1)- GARCH(1,1)	AR(3)- GARCH(1,1)	AR(3)- TARCH(1,1)	AR(3)- TARCH(1,1)*
Mean equation						
C	1.018 (0.001)	1.016 (0.002)	1.016 (0.002)	1.016 (0.002)	1.018 (0.002)	1.017 (0.002)
AR(1)	0.131 (0.012)	0.897 (0.058)	0.925 (0.012)	0.795 (0.051)	0.554 (0.042)	0.868 (0.040)
AR(2)	0.497 (0.017)	0.030 (0.061)	-	-0.274 (0.060)	0.085 (0.069)	-0.313 (0.057)
AR(3)	-	-	-	0.431 (0.045)	0.301 (0.055)	0.394 (0.048)
Variance equation						
C	$9.62 \times 10^{-6}$ ( $0.77 \times 10^{-6}$ )	$1.43 \times 10^{-7}$ ( $0.57 \times 10^{-7}$ )	$1.47 \times 10^{-7}$ ( $0.59 \times 10^{-7}$ )	$0.566 \times 10^{-7}$ ( $0.50 \times 10^{-7}$ )	$3.61 \times 10^{-7}$ ( $1.41 \times 10^{-7}$ )	$2.27 \times 10^{-7}$ ( $0.438 \times 10^{-7}$ )
ARCH(1)	1.565 (0.140)	0.337 (0.036)	0.344 (0.036)	0.359 (0.039)	0.309 (0.035)	0.470 (0.057)
( $\epsilon_t < 0$ ) ·ARCH(1)	-	-	-	-	-0.165 (0.067)	-0.492 (0.052)
GARCH(1)	-	0.745 (0.025)	0.741 (0.026)	0.756 (0.024)	0.745 (0.032)	0.769 (0.030)
Spred(-1)	-	-	-	-	-	$6.29 \times 10^{-5}$ ( $2.69 \times 10^{-5}$ )
Information criteria						
LogL	1722.634	1930.345	1933.407	1961.246	1946.680	1987.802
Akaike	-7.324665	-8.206162	-8.205988	-8.351480	-8.284956	-8.456421
Schwarz	-7.280416	-8.153063	-8.161810	-8.289430	-8.214042	-8.376643

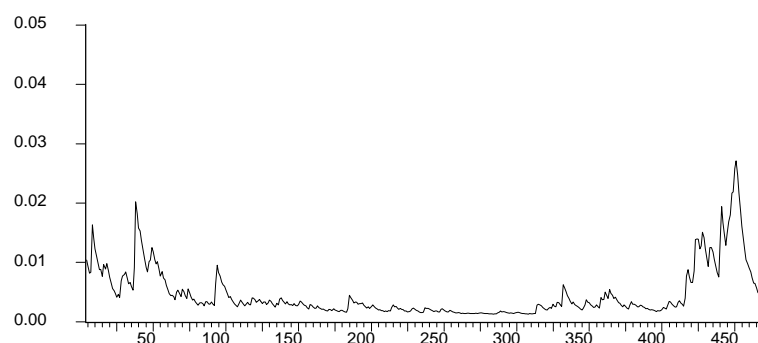
The simple model AR(3) - TARCH(1,1), being the third best according to the Information Criteria, is more preferable for us, since it reflects such important peculiarity of the returns behavior as the influence of “good/bad” news. The residual values less than zero correspond to the events when the model overestimates a returns growth (underrates their reduction) in comparison with the actual observations. Overestimation or underestimation of the differences can be an evidence of an influence of an inflow of “good” news at the current moment. Negative values of the estimate near the term ( $\epsilon_t < 0$ )·ARCH(1) means that "good" news have the leverage effect, reducing variance of the rate differences. Let us see whether we have managed by means of this model to explain strong hikes of volatility which become the reason of the pure expectations hypothesis not holding. Fig. 8 presents the residuals from the selected AR(3) - TARCH(1,1) model.



**Fig. 8.** Residuals from the AR(3)–TARCH(1,1) model.



**Fig. 9.** Standardized residuals.



**Fig. 10.** Conditional heteroskedasticity.

We can see that this model, though takes into account the characteristic of the dynamics of the financial variable and the influence of bad/good news, not well enough describes the raw data at certain moments of time. At the ends of the time interval under consideration the conditional volatility turns out to be significantly above that in the middle. This makes one think of a possibility of different regimes for the variance at the middle, initial and final intervals

Instability of a model is sometimes defined as switching of regression equations from one sub period (or regime) to another. For testing the structured changes in the cases when the dates between the regimes are known, the F-test, suggested by Chow (1960), can be used. The Chow test performed for the break dates  $t=100$  and  $t=415$  for our series of 470 observations for the simple AR(2) model speaks in favor of the presence of structural breaks.

Chow Breakpoint Test: $t_1 = 100, t_2 = 415$			
F-statistic	4.45945	Probability	0.000211
Log likelihood ratio	26.51681	Probability	0.000178

In our case we only have an approximate idea about the dates of shifts. However, so different behavior of returns during different time periods is indicative of the possibility of different regimes, i.e. of different means and variances in different regimes of the time series.

## 7. MODELING OF CONDITIONAL VARIANCE WITH SEVERAL REGIMES

### 7.1. Model with several regimes of unconditional volatility

Our purpose in this section is the construction of a model with conditional heteroskedasticity where the conditional variance would be different at different time intervals. Above it was repeatedly mentioned that in the behavior of some daily data on 1-month GKO profitability during the examined period one can trace three periods where volatility behaved differently: the initial period (approximately till October 1996), the medium period, and the period from May 1998 up to the end of the sample. In the first and third period the conditional variance is obviously higher, than in the second, which historically was caused by the uncertainty associated with the presidential elections in June 1996 and the inevitable approach of the crisis as a result of the short-sighted management of the state debt. To model such situation one could use the ARCH model with dummy variables in the equation for the conditional variance, allowing one to estimate the variance on each interval. For example, below the GARCH (1,1) model is written out, where the equation for the conditional variance, the dummy variables allow one to obtain an intercept for each of the three intervals.

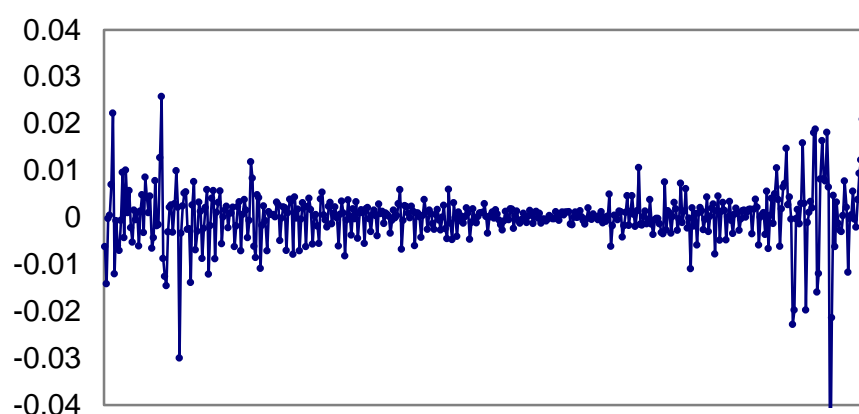
$$\begin{aligned}
 y_t &= \mu + \alpha_1 y_{t-1} + \varepsilon_t \\
 \sigma_t^2 &= \omega + \beta \varepsilon_{t-1}^2 + \gamma \sigma_{t-1}^2 + \delta_1 d_{1t} + \delta_2 d_{2t} \\
 d_{1t} &= \begin{cases} 1 & t \leq t_1 \\ 0 & t > t_1 \end{cases} \quad d_{2t} = \begin{cases} 0 & t < t_2 \\ 1 & t \geq t_2 \end{cases} \quad t_1 < t_2
 \end{aligned}$$



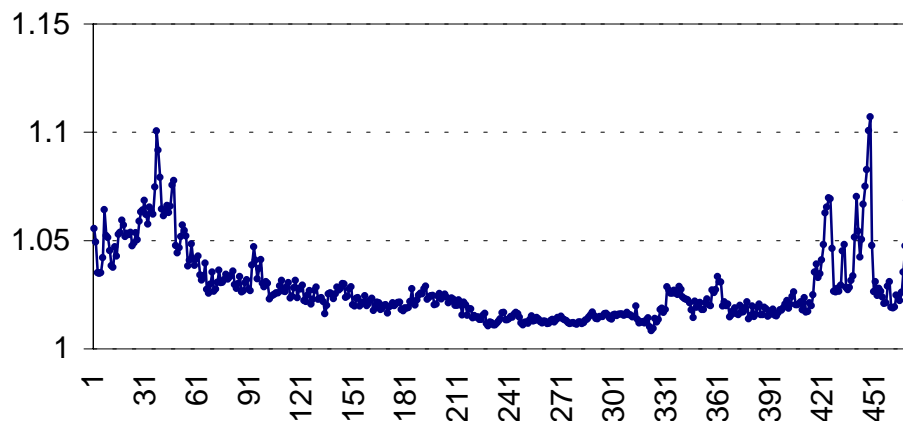
Generally speaking, such modeling in the literature is investigated insufficiently fully. The considered model is difficult to classify, as the threshold model (TARCH) in its classical meaning presumes that the dispersion varies when a certain random variable (namely, the error) takes values more or less than a threshold (of zero). Here this variable is the moment in time, and we decide whether it is more or less the threshold. Similar models, without dummies, but with different coefficients in the equation depending on whether we are to the left or to the right of the time threshold value, are described in the literature, but only for AR, and, accordingly, switches there occur only in the mean equation.

The main problem when attempting to estimate our model is the stage of finding the estimates of the threshold values  $t$  and properties of these estimates. At present, the literature does include papers about stationarity in series with structural breaks, (for example, Perron and Vogelsand, 1992), but no universal algorithm we could use for testing structural breaks in conditional volatility is developed yet. The construction of an appropriate test, confidence intervals and properties of estimators for the threshold values  $t$  is not a trivial problem. We will so far concentrate at the second stage – the estimation of models under the assumption that the moments  $t_1$  и  $t_2$  are known or already found.

The choice of time moments  $t_1$  and  $t_2$  at the present stage was made by a visual analysis of the time series for differences of 1-month returns  $\Delta y_t = y_{t+1} - y_t$  (See Fig. 11) and the behavior of the 1-month return itself. (Fig. 12). We decided on the time moments 07.10.1996 for  $t_1$  and 15.05.1998 for  $t_2$ . Generally speaking, under a more thorough look at the behavior of the differences of returns one may see five regimes of volatility – very high, then more or less tranquil period, absolutely tranquil period and again more or less tranquil, then high again. But, as the examined period is rather short, we will concentrate on the three periods.



**Fig. 11.** Daily differences of 1-month returns GKO.



**Fig. 12.** 1-month returns GKO.

We considered several different specifications, the best of which proved to be the GARCH (1,1) model with autoregression of the second order and dummy variables not only in the variance equation, but also in the mean equation (Table 4).

**Table 4.**

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.163042	0.014698	11.09303	0.0000
GKO(-1)	0.967807	0.039968	24.21479	0.0000
GKO(-2)	-0.128080	0.025777	-4.968801	0.0000
D1	0.003861	0.000721	5.356440	0.0000
D2	0.004636	0.001278	3.628419	0.0003
Variance Equation				
C	1.65E-06	5.78E-07	2.852829	0.0043
ARCH(1)	0.150882	0.033718	4.474800	0.0000
GARCH(1)	0.600459	0.094012	6.387072	0.0000
D1	1.03E-05	4.00E-06	2.575519	0.0100
D2	3.66E-05	1.20E-05	3.058924	0.0022

It is interesting, that after the introduction of different regimes for the conditional variance on each of the three intervals, the coefficient at  $(\varepsilon_t < 0) \cdot \text{ARCH}(1)$  has ceased to be significant when we tried to estimate the conventional TARCH model, with the threshold value for the errors  $\varepsilon_t = 0$ , although such model estimated on the whole interval showed a good output. So we see that the dummy variables really bring a big contribution to an explanation of the dynamics of 1-month GKO profitability.

Strictly speaking, the estimated model is rather unconditionally heteroskedastic than conditionally heteroskedastic since the dummy variables influence only the unconditional variance, and the coefficients at the square of the previous error value and at the previous value of the conditional variance remain constant during all three subperiods. Since the series at hand is stationary on the three subperiods, we can calculate explicitly using the obtained estimates the unconditional variances for each of them according to the following formulas:

$$\sigma_1^2 = \frac{\omega + \delta_1}{1 - \beta - \gamma} = 0.000048; \quad \sigma_{II}^2 = \frac{\omega}{1 - \beta - \gamma} = 0.0000066 \quad \text{and} \quad \sigma_{III}^2 = \frac{\omega + \delta_2}{1 - \beta - \gamma} = 0.0001538.$$

Thus we have obtained a conditionally heteroskedastic model where the unconditional variance is different on each of the three subperiods: the smallest variance on the middle period, second-largest on the earliest one and the biggest - on the latest one. This exactly corresponds to the quietest middle period, higher volatility on the earliest period (prior to elections) and to the pre-crisis period with the highest volatility. In the same way the average return on each of the three subperiods – the lowest on the middle interval, higher on the first, and the highest on the last, according to the risk premium required by investors for different levels of uncertainty on each interval.

## 7.2. Model with several regimes of conditional volatility

In a more traditional sense a conditionally heteroskedastic model with dummy variables that influence the conditional variance looks as follows:

$$\begin{aligned} y_t &= \mu + \alpha_1 y_{t-1} + \varepsilon_t \\ \sigma_t^2 &= \omega + (\beta + \delta_1 d_{1t} + \delta_2 d_{2t}) \varepsilon_{t-1}^2 + (\gamma + \rho_1 d_{1t} + \rho_2 d_{2t}) \sigma_{t-1}^2 \end{aligned} \quad (9)$$

$$d_{1t} = \begin{cases} 1 & t \leq t_1 \\ 0 & t > t_1 \end{cases} \quad d_{2t} = \begin{cases} 0 & t < t_2 \\ 1 & t \geq t_2 \end{cases} \quad t_1 < t_2$$

That is, when switching from one subinterval to another the equation for the conditional variance changes the coefficients at  $\sigma_{t-1}^2$  and  $\varepsilon_{t-1}^2$ , although in a full sense traditional is consideration of conditionally heteroskedastic models in which a change of the coefficient at  $\varepsilon_{t-1}^2$  occurs when the error itself changes its sign. We shall try to evaluate how such variant of modeling of the conditional variance fits our data. Estimation and testing of this model, in comparison with the previous one, is more laborious as more parameters appear. Therefore we shall look first of all whether it suits us judging by some auxiliary informal considerations.

To this end, let us estimate first the model for the conditional mean in the same way as in the model (8) (AR(2) with a constant and two dummy variables  $d_{1t}$  and  $d_{2t}$ ), on the whole time period. Accordingly, the estimation results for the conditional mean will be the same, as in the first part of Table 4. After that, we will fit GARCH(1,1) to the residuals, but now separately on each of the three subperiods. Certainly, this is not the same as estimation of the entire model (9) on the whole time period, but it nevertheless allows one informal evidence about time stability of the model which is

extremely laborious to get via formal testing. So, for each of the subperiods we obtained the following results:

**Table 5.**

	Variance Equation (for the first period)			
	Coefficient	Std. Error	z-Statistic	Prob.
C	4.97E-06	4.20E-06	1.184561	0.2362
ARCH(1)	0.104206	0.045936	2.268485	0.0233
GARCH(1)	0.796180	0.090568	8.790952	0.0000

	Variance Equation (for the second period)			
	Coefficient	Std. Error	z-Statistic	Prob.
C	8.31E-08	3.16E-08	2.626315	0.0086
ARCH(1)	0.128637	0.033165	3.878754	0.0001
GARCH(1)	0.870912	0.029799	29.22663	0.0000

	Variance Equation (for the third period)			
	Coefficient	Std. Error	z-Statistic	Prob.
C	4.00E-05	1.93E-05	2.078175	0.0377
ARCH(1)	0.777448	0.443986	1.751067	0.0799
GARCH(1)	0.149195	0.265186	0.562605	0.5737

We can make the following conclusions from the given results. For the first two subperiods the estimated GARCH coefficients are practically indistinguishable (relative to two standard errors) with significance level 5%. On the third subperiod these coefficients differ strongly from those for the first two subperiods, but these estimates are hardly trustable in view of very small values of z-statistics and a small number of observations within this interval (only 57 observations).

Recognizing that estimates of the coefficients at the squared previous value of the conditional variance and the squared error in the equation for the conditional variance are practically indistinguishable for the first two subperiods and have bad statistical properties for the third one, we conclude that estimating the model (9) for our data is not reasonable. Thus, the examined series of daily returns on one-month GKO exhibits different behaviour on different subintervals of unconditional heteroskedasticity, instead of the conditional one. The previous values of squared errors and of conditional variance bring identical contributions to the formation of today's conditional variance on all the three subperiods, different appears to be the intercept, that is, the unconditional variance.

## 8. POLICY CONCLUSIONS

This work studies the GKO market in the period of 1996-1998, the period from the past but still representing interest for a researcher. Using the data from daily secondary GKO trades first the yield curve was constructed, and then an attempt to explain its dynamics was undertaken with the help of the pure expectations hypothesis. The hypothesis was tested in two forms. The second form meaning that the majority of investors at the Russian market were aiming at long-term investment, has not received any confirmation. This is no surprising as the state itself did not focus on long-term borrowing. The first form meaning the prevalence at the market of speculators maximizing short-term profits, turned out to be true for the time horizon of 6 months. For shorter terms (2 and 4 months) a tendency has come to light that is opposite to the one stated by the hypothesis, namely, that at the Russian GKO market during the examined period a high spread in returns in the short term was leading to a fall in long term returns. The same result was repeatedly obtained in the literature when testing the pure expectations hypothesis at financial markets of other countries. Let us list the main reasons of the obtained results:

- Political instability, bad situation with the budget deficit (since 1997, after the fall in energy and metals prices, there was a need of ruble emission and appropriate fall of the ruble exchange rate. Its artificial restraint resulted only in the refusal of the government to carry out its obligations instead of gradual adaptation to new economic conditions.). The consequence of it was illiquidity of long papers, instability of the GKO market relative to the currency market.
- A big share of foreign investors participating in trades, in contrast to Russian ones, was considering the Russian market as one of high risk. After the removal of all restrictions on participation of non-residents in trades and the permission of making foreign currency forward contracts the GKO market and the associated market of foreign currency began to serve not so much for hedging operations as for short-term speculative operations. As a result of this the capital was quickly moving among investments in various GKO releases or was being converted to the foreign currency while changing the market structure and destabilizing the market.
- Actions of few large agents in the market, who formed more than half of trade volumes and possessed more information than small investors, made their contribution to the inefficiency of the GKO market.

All this explains frequent fluctuation of the market spread and the slope of the yield curve which were giving big opportunities for arbitrage operations. Slightest ambiguous information of political or economic character from inside or outside instantly made longer papers illiquid, overturning the slope of the yield curve from normally positive to negative. The obtained estimate of factor  $\beta(N)$  close to  $-1$  confirms this conclusion: a high spread in the short run leads not to an increase of returns on long papers as should be in a stable developed market, but is perceived as a disturbing signal, and acts upon long rates in the opposite direction, as a result the yield curve becoming negatively sloped.

An attempt to improve the results of the hypothesis testing by examining only a part of the time period having eliminated the initial and final subperiods rich on various sorts political and economic shocks was not successful. The reason was the impact of the world financial crises in addition to all factors listed above.

As the rationality of expectations has not been confirmed at the Russian market, some conclusions about investor behavior can be made from studying the dynamics of volatility of the daily data on one-month GKO. However from consideration of several models for conditional heteroskedasticity and their properties it was found out that even they do not fully enough describe different behaviour of returns observed on different subperiods. Two alternative models of conditional heteroskedasticity therefore were considered that allow one to estimate the conditional variance on each of the subperiods with the help of dummy variables in the equation for the conditional variance. As a result we chose a most adequate for the data model with conditional heteroskedasticity exhibiting varying unconditional variance for the three subperiods was selected: on each of them the equation for the conditional variance has own intercept, but it is unreasonable to believe that it contains different coefficients at the ARCH and GARCH terms on different subperiods. Hence, an average return on each of three subperiods – the lowest on the middle interval, higher on the first and the highest on the last – was evaluated, corresponding to the risk premium required by investors for different levels of uncertainty (a unconditional variance) on each interval.

The main lesson that was important to learn from this experience – the state securities themselves cannot solve such countrywide problems as high inflation or bad balance of payments. Only under coordination with inflation rates, budget deficit, global deposit rates are they capable to improve national economy; otherwise they turn to a speculative tool.

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